Kinematics and Dynamics of Particles

Newton's Laws of Motion

Newtonian mechanics only valid when $v << c$ and w/r to an inertial reference frame

↳ axes have no translation or rotation in space

2^{nd}: $\boxed{F = \frac{dp}{dt} = \frac{d(mv)}{dt} = ma}$

3^{rd}: every action has opposed equal reaction

1^{st} Law: every body preserves state of rest or uniform motion unless compelled to change state by external force

Orbital Mechanics

Micro-gravity: An object in orbit is not weightless

weight due to gravity:

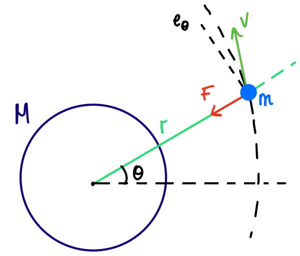
$a = \frac{GM}{r^2} = \boxed{\frac{GM}{(R+h)^2}}$

$\boxed{\frac{a}{g} = \left(\frac{R}{R+h}\right)^2}$

gravity constant at altitude above earth as % of g

$\Sigma F_r: \; -G\frac{mM}{r^2} = m(\ddot{r} - r\dot{\theta}^2)$

$\Sigma F_\theta: \; 0 = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$



For circular orbit:

↘ constant r

$\boxed{\dot{\theta}^2 = \frac{GM}{r^3}}$

with $\dot{\theta} = \frac{v}{r} \rightarrow \boxed{v = \sqrt{\frac{GM}{r}}}$

$\boxed{T = \frac{2\pi r}{v} = 2\pi\sqrt{\frac{r^3}{GM}}}$

$F = G\frac{Mm}{r^2}$ $\quad G = 6.67 \times 10^{-11}$

Acceleration

Constant (SUVAT)

- derived from integrals

$v(t) = u + at$

$v^2 = v_0^2 + 2a\Delta s$

$s(t) = s_0 + ut + \frac{1}{2}at^2$

CHECK WITH SUVAT (EST.)

non-constant

$a\,ds = v\,dv$

↑ 'acceleration w/r to displacement'

a as function of time

$a = \frac{dv}{dt} \rightarrow f(t) = \frac{dv}{dt}$

$\int f(t)\,dt = \int dv \rightarrow \boxed{v(t) = v_0 + \int f(t)\,dt}$

$\rightarrow \boxed{s(t) = s_0 + \int v(t)\,dt}$

a as function of displacement

$f(s) = a$ & $a\,ds = v\,dv$

$\rightarrow \int f(s)\,ds = \int v\,dv$

$= \boxed{v^2 = v_0^2 + 2\int f(s)\,ds}$

with $v = g(s)$ ↖ another function of displacement

$\boxed{t = \int \frac{ds}{g(s)}}$

a as function of velocity

$a = \frac{dv}{dt} \longrightarrow f(v) = \frac{dv}{dt}$

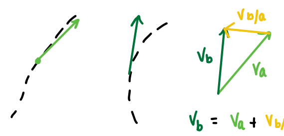
$\int dt = \int \frac{dv}{f(v)} \rightarrow \boxed{t = \int \frac{dv}{f(v)}}$

$\rightarrow$ using $a\,ds = v\,dv$

$\int \frac{v}{f(v)}\,dv = \int ds \rightarrow \boxed{s(v) = s_0 + \int \frac{v}{f(v)}\,dv}$

Relative Motion

Velocity



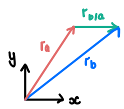
$v_b = v_a + v_{b/a}$

for observer at v_a, particle b appears to move with velocity $v_{b/a}$

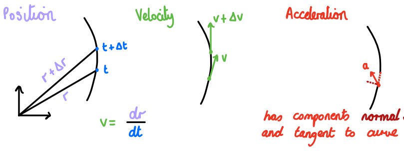
↖ relative velocity

Displacement

$r_B = r_a + r_{b/a}$



Planar Curvilinear Motion



$v = \frac{dr}{dt}$

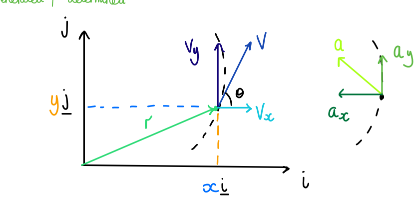
has components normal and tangent to curve

↖ points towards local centre of curvature

$a = \frac{dv}{dt}$

Cartesian

- Useful when x & y components of acceleration are independently generated / determined



$r = xi + yj$

$v = \dot{r} = \dot{x}i + \dot{y}j$

$a = \dot{v} = \ddot{r} = \ddot{x}i + \ddot{y}j$

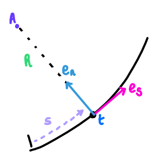
$a^2 = a_x^2 + a_y^2$

$v^2 = v_x^2 + v_y^2$

$\tan\theta = \frac{v_y}{v_x}$

Intrinsic

- Natural, often most convenient



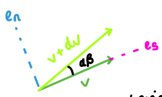
e_s = local tangent

e_n = local normal → +ve towards local centre = A

R = radius of curvature

Velocity along tangent:

$v_s = \frac{ds}{dt} = \dot{s}$ $\quad v_n = 0$



using $\cos d\beta \approx 1$, $\sin d\beta \approx d\beta$

$a_s = \frac{(v+dv)\cos d\beta - v}{dt} = \frac{dv}{dt} = \dot{v} = \ddot{s}$

$a_n = \frac{(v+dv)\sin d\beta - 0}{dt} = \frac{v\,d\beta}{dt} = \frac{v}{dt}\frac{ds}{dR}$

$= \frac{v}{R}\frac{ds}{dt} = \frac{v^2}{R}$

dβdv neglected $\quad ds = R\,d\beta$ → RA triangle

centripetal acceleration

↳ +ve in direction of centre

∴ Σa must lie inside curve

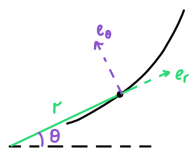
Circular Motion

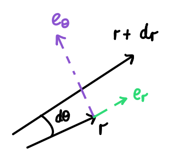
$v = r\dot{\theta}$

$a_n = \frac{v^2}{R} = r\dot{\theta}^2 = v\dot{\theta}$

$a_t = \dot{v} = r\ddot{\theta}$

Polar





$v_r = \frac{(r+dr)\cos d\theta - r}{dt} = \frac{dr}{dt} = \dot{r}$

$v_\theta = \frac{(r+dr)\sin d\theta}{dt} = \frac{r\,d\theta}{dt} = r\dot{\theta}$

$a_r = \ddot{r} - r\dot{\theta}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

↑ Coriolis term